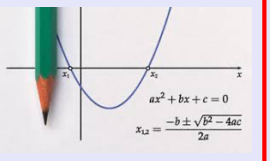
Math 125

Spring 2022

Lecture 5



Class QZ 3

Write $3x-5y=10$ (Standard) in Slope-Int. (y = mx + b) Form, then graph. Give slope & Y-Int. Clearly.

$3x-5y=10$

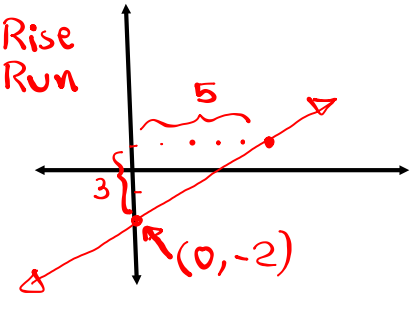
$-5y=-3x+10$

$y=\frac{-3}{-5}x+\frac{10}{-5}$

$y=\frac{3}{5}x-2$

Slope $m=\frac{3}{5}$ $\frac{\text{Rise}}{\text{Run}}$

Y-Int $(0,-2)$

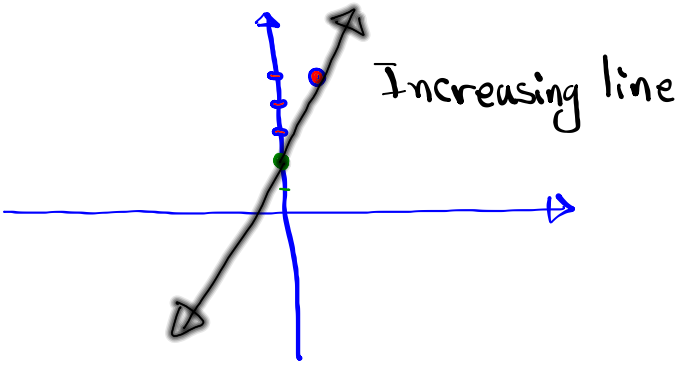


Consider $y = 3x + 2$

$y = mx + b$

$m = 3 = \frac{3}{1}$ Rise / Run

Y-Int (0,2)



Increasing line

Consider $y = -2x$

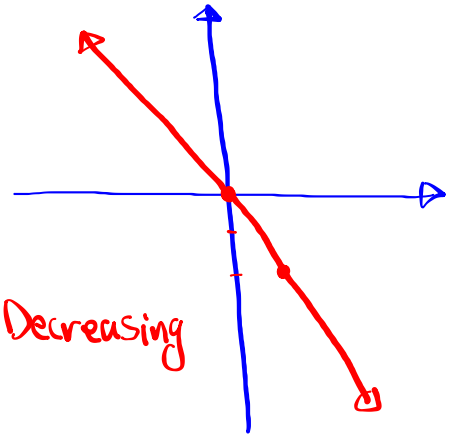
$y = mx + b$

1) Slope $m = -2 = \frac{-2}{1}$ Rise / Run

2) Y-Int (0,0)

3) Graph

4) Discuss increasing/Decreasing



Decreasing

$f(x) = 3x + 5$ Find

$g(x) = x - 5$

1) $f(0) = 3(0) + 5$
$= 0 + 5$
$= \boxed{5}$

2) $g(0) = 0 - 5$
$= \boxed{-5}$

3) $f(0) + g(0) = 5 + (-5) = \boxed{0}$

4) $(f+g)(x) = f(x) + g(x)$
$= 3x + 5 + x - 5 = \boxed{4x}$

5) $(f-g)(x) = f(x) - g(x)$
$= 3x + 5 - (x - 5)$
$= 3x + 5 - x + 5 = \boxed{2x + 10}$

6) $(f \cdot g)(x) = f(x) \cdot g(x)$
$= (3x+5)(x-5) = 3x^2 - 15x + 5x - 25$
Foil
$= \boxed{3x^2 - 10x - 25}$

7) $(f/g)(x) = \dfrac{f(x)}{g(x)}$; $g(x) \neq 0$

$= \boxed{\dfrac{3x+5}{x-5}}$; $\boxed{\begin{array}{l} x - 5 \neq 0 \\ x \neq 5 \end{array}}$

Given $f(x) = 2x - 3$, $g(x) = 2x + 3$

Find

1) $f(2) = 2(2) - 3$
$= 4 - 3$
$= \boxed{1}$

2) $g(-2) = 2(-2) + 3$
$= -4 + 3$
$= \boxed{-1}$

3) $(f + g)(x)$
$= f(x) + g(x)$
$= 2x - 3 + 2x + 3$
$= \boxed{4x}$

4) $(f - g)(x)$
$= f(x) - g(x)$
$= 2x - 3 - (2x + 3)$
$= 2x - 3 - 2x - 3$
$= \boxed{-6}$

5) $(f \cdot g)(x) = f(x) \cdot g(x)$
$= (2x - 3)(2x + 3) = 4x^2 + 6x - 6x - 9$
$= \boxed{4x^2 - 9}$

Foil

6) $(f/g)(x)$
$= \dfrac{f(x)}{g(x)}$; $g(x) \neq 0$
$= \boxed{\dfrac{2x-3}{2x+3} \; ; \; 2x + 3 \neq 0, \; 2x \neq -3, \; x \neq \dfrac{-3}{2}}$

Piece-wise Function

$$f(x)=\begin{cases} -4x & \text{if } x<0 \\ x^2-4 & \text{if } x\geq 0 \end{cases}$$

Find

1) $f(-2)=-4(-2)=\boxed{8}$

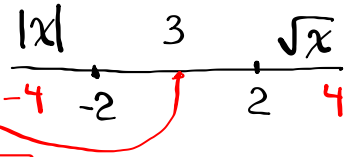
2) $f(0)=0^2-4$
$=0-4$
$=\boxed{-4}$

3) $f(2)=(2)^2-4=4-4=\boxed{0}$

$$g(x)=\begin{cases} |x| & \text{if } x<-2 \\ 3 & \text{if } -2\leq x\leq 2 \\ \sqrt{x} & \text{if } x>2 \end{cases}$$

Find

1) $f(-4)=|-4|=\boxed{4}$



2) $f(-2)=\boxed{3}$

3) $f(0)=\boxed{3}$

4) $f(2)=\boxed{3}$

5) $f(4)=\sqrt{4}=\boxed{2}$

x	2	0	3	5
$f(x)$	5	3	2	0
$g(x)$	0	2	3	2

$f(0) = \boxed{3}$ 2) $g(2) = \boxed{0}$

$(f+g)(2) = f(2)+g(2) = 5+0 = \boxed{5}$

$(f-g)(0) = f(0)-g(0) = 3-2 = \boxed{1}$

$(f \cdot g)(5) = f(5) \cdot g(5) = 0 \cdot 2 = \boxed{0}$

$(f/g)(2) = \frac{f(2)}{g(2)} = \frac{5}{0}$ undefined

$(g/f)(3) = \frac{g(3)}{f(3)} = \boxed{\frac{3}{2}}$

$(g/f)(5) = \frac{g(5)}{f(5)} = \frac{2}{0}$ undefined

Find the domain:

1) $f(x) = \frac{x-8}{x+2}$ Deno. $\neq 0$

$x+2 \neq 0$

$\boxed{x \neq -2}$

All Reals except -2

2) $g(x) = \frac{3x+5}{x-4}$

All Reals except 4

Deno. $\neq 0$

$x-4 \neq 0$

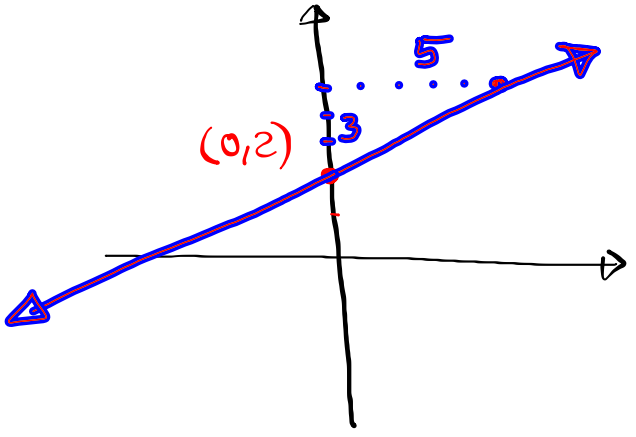
$x \neq 4$

3) $h(x) = \frac{x+10}{x}$ Deno. $\neq 0$

$x \neq 0$

All Reals except 0

Draw a line with Y-Int at (0,2) and slope $\frac{3}{5}$.



Slope-Int Form

$y = mx + b$

$y = \frac{3}{5}x + 2$

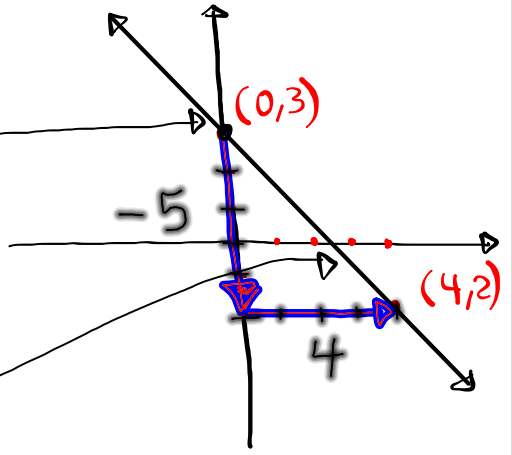
If we replace y with $f(x)$, we get

$\boxed{f(x) = \frac{3}{5}x + 2}$ Linear Function

Draw a line that contains (0,3) and (4,-2).

$m = \frac{-5}{4}$

Y-Int (0,3)



Slope-Int Form

$y = mx + b$

$y = \frac{-5}{4}x + 3$

Replace with $f(x)$, $\boxed{f(x) = \frac{-5}{4}x + 3}$ Linear Function.

Algebra Review:

1) Factor $3x^2 - 2x - 5 =$

$(3x + 1)(x - 5)$

$3x(-5) + 1x = -15x + 1x = -14x$

$(3x - 5)(x + 1)$

$3x(-1) + 5(x) = -3x + 5x = 2x$

$3x(1) - 5(x) = 3x - 5x = -2x$

$3x^2 - 2x - 5 = (3x - 5)(x + 1)$

Solve and graph

$2x + 8 \leq 5x - 4$

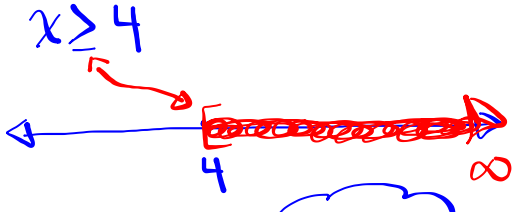
Hint: Isolate x on the Left-hand Side.

$2x - 5x \leq -4 - 8$

$-3x \leq -12$

$\frac{-3}{-3}x \geq \frac{-12}{-3}$

$x \geq 4$



SG 3 ✓

Interval notation $[4, \infty)$

Set-Builder notation $\{x \mid x \geq 4\}$

Set: A collection of objects

Set notation { }

Ex: $A = \{2, 4, 6, 8\}$ $B = \{4, 5, 6\}$

Operations with Sets

1) Union $A \cup B$ → all elements of A, B, or both

2) Intersection $A \cap B$ → only the common elements of A and B.

$A \cup B = \{2, 4, 6, 8, 4, 5, 6\} = \{2, 4, 6, 8, 5\}$

$= \{2, 4, 5, 6, 8\}$ ✓

$A \cap B = \{4, 6\}$

$A = \{1, 3, 5, 7, 9\}$ $B = \{0, 2, 4, 6, 8\}$

$A \cup B = \{1, 3, 5, 7, 9, 0, 2, 4, 6, 8\}$

$= \{0, 1, 2, \ldots, 9\}$

$A \cap B = \{\ \} = \emptyset$ empty set

NO Common element

Compounded Inequality

Hint: Isolate x in the middle

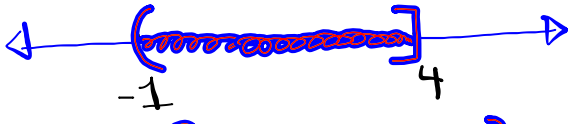
$$-2 < 3x + 1 \leq 13$$

$$-2 - 1 < 3x + 1 - 1 \leq 13 - 1$$

$$-3 < 3x \leq 12$$

$$\frac{-3}{3} < \frac{3}{3}x \leq \frac{12}{3}$$

$$-1 < x \leq 4$$



I.N. $(-1, 4]$, S.B.N. $\{x \mid -1 < x \leq 4\}$

Solve

Hint: Isolate x in the middle.

$$-5 < -2x - 5 \leq 15$$

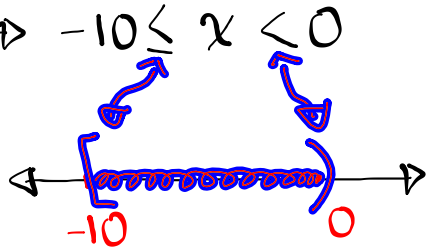
$$-5 + 5 < -2x - 5 + 5 \leq 15 + 5$$

$$0 < -2x \leq 20$$

$$\frac{0}{-2} > \frac{-2}{-2}x \geq \frac{20}{-2}$$

$$0 > x \geq -10$$

$-10 \leq x < 0$



I.N. $[-10, 0)$

S.B.N. $\{x \mid -10 \leq x < 0\}$

Find the domain $f(x) = \dfrac{2x-3}{x^2-100}$

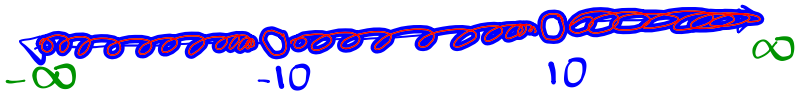
1) Deno. $\neq 0$

$x^2-100 \neq 0$

$x^2 \neq 100$

$x \neq \pm 10$

All Reals except ± 10



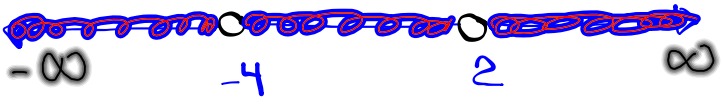
I.N. $(-\infty, -10) \cup (-10, 10) \cup (10, \infty)$

Find the domain for $f(x) = \dfrac{x}{(x-2)(x+4)}$.

$(x-2)(x+4) \neq 0$

$x-2 \neq 0$ $\quad x+4 \neq 0$

$x \neq 2$ $\quad x \neq -4$



I.N.

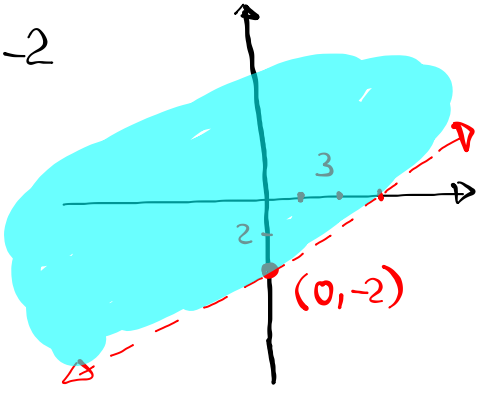
$(-\infty, -4) \cup (-4, 2) \cup (2, \infty)$

Graph and shade in rectangular Coordinate System

shade above

$y > \frac{2}{3}x - 2$ $\quad y = \frac{2}{3}x - 2$

dotted line

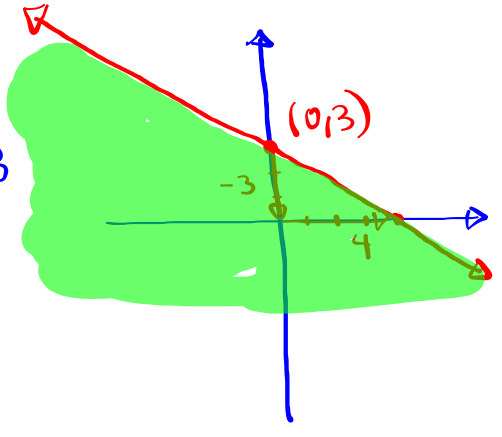


Graph and shade

shade Below

$y \leq \frac{-3}{4}x + 3$ $\quad y = \frac{-3}{4}x + 3$

Solid line



You can start SG 4

Class QZ 4

$f(x) = 3x + 4$ $\quad$ $g(x) = 3x - 4$

Find

1) $(f+g)(x) = f(x) + g(x)$
$= 3x + 4 + 3x - 4$
$= \boxed{6x}$

2) $(f-g)(x)$
$= f(x) - g(x)$
$= 3x + 4 - (3x - 4)$
$= 3x + 4 - 3x + 4 = \boxed{8}$

3) $(f \cdot g)(x)$
$= f(x) \cdot g(x)$
$= (3x+4)(3x-4) = 9x^2 - 12x + 12x - 16 = \boxed{9x^2 - 16}$